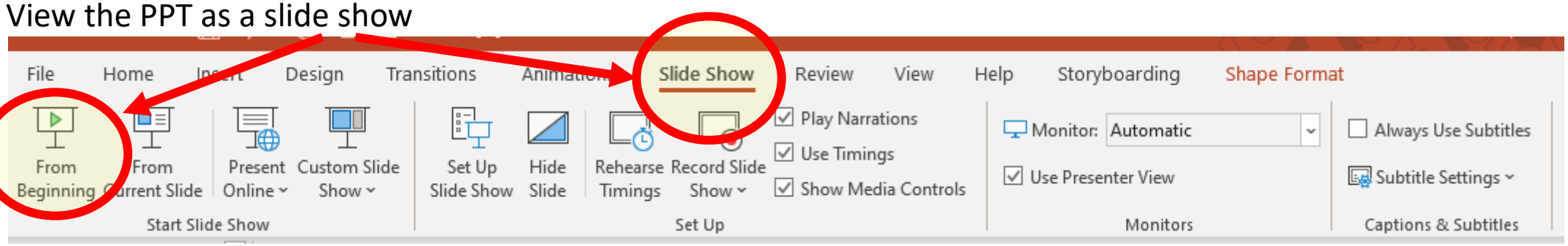
How to best use these slides…

- View the PPT as a slide show



- Then click through every step
  - Mouse clicks will advance the slide show
  - Left/right arrow keys move forward/backward
  - Mouse wheel scrolling moves forward/backward
- When a question is posed, stop and think it through, try to answer it yourself before clicking
- If you have questions, use PS discussion boards, email me, and/or visit us in a Teams class session!

LESSON 7.3a

Multiplying Rational Expressions

Today you will:

- Simplify rational expressions
- Multiply rational expressions
- Practice using English to describe math processes and equations

Core Vocabulary:

- Rational expression, p. 376
- Simplified form of a rational expression, p. 376

Prior:

- Fractions and fraction arithmetic
- Polynomials
- Domain
- Equivalent expressions
- Reciprocal

Today we are going to multiply *Rational Expressions*

Tomorrow we will divide them…

- Heads up…we will turn our division problems into multiplication (reciprocal)
- So getting our multiplication skills down pat is ***important!***

But first we need to:

1. Figure out what a ***rational expression*** is:
    - One polynomial divided by another
    - In other words, a fraction with a polynomial on top and another on the bottom
    - $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are both non-zero polynomials
    - Example: $\frac{3x^2+6}{9x-12}$
    - Note there is no = sign. Why? Because this is an ***EXPRESSION*** not an equation. ☺
2. Since a rational expression is basically a fraction, we also review our fraction arithmetic rules!

Fraction Arithmetic

Okay settle down, we can do this…

We are going to be multiplying so let's focus on how to multiply fractions

1. Simplify simplify simplify … did I mention simplify? No? Okay … simplify
   - Divide out (people often say cancel) common factors
   - Note I said simplify ***FACTORS*** not terms
   - Factors means **product** which means things **multiplied** together
   - Example: simplify $\frac{15}{65} = \frac{3 \cdot \cancel{5}}{13 \cdot \cancel{5}} = \frac{3}{13}$
   - Example: simplify $\frac{4\cancel{(x+3)}}{(x+3)\cancel{(x+3)}} = \frac{4}{x+3}$
   - We ***CANNOT*** do the following:

     DON'T DO THIS example: $\frac{x+3}{x}$ … you ***CANNOT*** divide out the x

     Why? We can only divide out things that are multiplied

     In the numerator x and 3 are terms ***NOT*** products

Note in both these cases we are ***DIVIDING*** things that are ***MULTIPLIED***

Fraction Arithmetic - Simplifying

- Note from the prior examples you may need to factor in order to simply.
- Let me say that again … you may need to ***factor a polynomial***. ☺
- This means you might want to go back and review our factoring lessons!
- Here are links to some of the key lessons and PowerPoints from Chapter 4:
  - Short chapter review (from our midterm preps)
  - L4.2c Polynomial Identities & Patterns
  - L4.4a Special Polynomial Factoring Patterns
  - L4.4b Factoring Polynomials by Grouping
  - L4.4c Factor Theorem

Okay!
Here we go!

Simplify $\frac{x^2 - 4x - 12}{x^2 - 4}$.

SOLUTION

$$\frac{x^2 - 4x - 12}{x^2 - 4} = \frac{(x + 2)(x - 6)}{(x + 2)(x - 2)}$$ Factor numerator and denominator.

$$= \frac{\cancel{(x + 2)}(x - 6)}{\cancel{(x + 2)}(x - 2)}$$ Divide out common factor.

$$= \frac{x - 6}{x - 2}, \; x \neq -2$$ Simplified form

COMMON ERROR

Do not divide out variable terms that are not factors.

$\frac{x - 6}{x - 2} \neq \frac{-6}{-2}$

The original expression is undefined when $x = -2$. To make the original and simplified expressions equivalent, restrict the domain of the simplified expression by excluding $x = -2$. Both expressions are undefined when $x = 2$, so it is not necessary to list it.

Fraction Arithmetic - Multiplying

1. Simplify each fraction
2. Multiply across
3. Simplify the result

If you want you can just multiply then simplify…

What does it mean to ***multiply across***?

- Numerator times numerator
- Denominator times denominator
- This does ***NOT***, I repeat ***NOT*** mean cross multiplication
- Multiply the top times the top
- …then the bottom times the bottom

Example: Find the product $\frac{2x}{6} \cdot \frac{9}{x^2}$

$$\frac{2x}{6} \cdot \frac{9}{x^2} = \frac{\cancel{2} \cdot x}{\cancel{2} \cdot 3} \cdot \frac{9}{x^2}$$ Simplify: factor

$$= \frac{x}{3} \cdot \frac{9}{x^2}$$ and cancel

$$= \frac{x \cdot 9}{3 \cdot x^2}$$ Multiply across

$$= \frac{\cancel{3} \cdot 3 \cdot \cancel{x}}{\cancel{3} \cdot x \cdot \cancel{x}}$$ Simplify: factor

$$= \frac{3}{x}$$ and cancel

Note: when I use the term “cancel” I really mean “divide common factors”

ANOTHER WAY

In Example 2, you can first simplify each rational expression, then multiply, and finally simplify the result.

$$\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$$
$$= \frac{4x^2}{y} \cdot \frac{7x^4y^2}{4y}$$
$$= \frac{\cancel{4} \cdot 7 \cdot x^6 \cdot \cancel{y} \cdot y}{\cancel{4} \cdot \cancel{y}}$$
$$= 7x^6y,\ x \neq 0,\ y \neq 0$$

Find the product $\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$.

Note when multiplying first then simplifying you end up having to refactor back to where you started!

SOLUTION

$$\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y} = \frac{56x^7y^4}{8xy^3}$$

Multiply numerators and denominators.

$$= \frac{\cancel{8} \cdot 7 \cdot \cancel{x} \cdot x^6 \cdot \cancel{y^3} \cdot y}{\cancel{8} \cdot \cancel{x} \cdot \cancel{y^3}}$$

Factor and divide out common factors.

$$= 7x^6y,\ x \neq 0,\ y \neq 0$$

Simplified form

Find the product $\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x}$.

SOLUTION

$$\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x} = \frac{3x(1 - x)}{(x - 1)(x + 5)} \cdot \frac{(x + 5)(x - 4)}{3x}$$

Factor so can divide out common factors in each.

$$= \frac{3x(-1)(x - 1)}{(x - 1)(x + 5)} \cdot \frac{(x + 5)(x - 4)}{3x}$$

Rewrite $1 - x$ as $(-1)(x - 1)$ and cancel

$$= \frac{3x(-1)(x + 5)(x - 4)}{(x + 5)(3x)}$$

Multiply numerators and denominators.

$$= \frac{3x(-1)(x - 1)(x + 5)(x - 4)}{(x - 1)(x + 5)(3x)}$$

Divide out common factors.

$$= -x + 4,\ x \neq -5,\ x \neq 0,\ x \neq 1$$

Simplified form

Check

X	Y1	Y2
−5	ERROR	9
−4	8	8
−3	7	7
−2	6	6
−1	5	5
0	ERROR	4
1	ERROR	3

X=−4

Check the simplified expression. Enter the original expression as y_1 and the simplified expression as y_2 in a graphing calculator. Then use the *table* feature to compare the values of the two expressions. The values of y_1 and y_2 are the same, except when $x = -5$, $x = 0$, and $x = 1$. So, when these values are excluded from the domain of the simplified expression, it is equivalent to the original expression.

Find the product $\frac{x+2}{x^3-27} \cdot (x^2+3x+9)$.

STUDY TIP

Notice that $x^2 + 3x + 9$ does not equal zero for any real value of x. So, no values must be excluded from the domain to make the simplified form equivalent to the original.

SOLUTION

CAN factor

CANNOT factor

$$\frac{x+2}{x^3-27} \cdot (x^2+3x+9) = \frac{x+2}{x^3-27} \cdot \frac{x^2+3x+9}{1}$$

Write polynomial as a rational expression.

$$= \frac{(x+2)(x^2+3x+9)}{(x-3)(x^2+3x+9)}$$

Multiply. Factor denominator.

$$= \frac{(x+2)\cancel{(x^2+3x+9)}}{(x-3)\cancel{(x^2+3x+9)}}$$

Divide out common factors.

$$= \frac{x+2}{x-3}$$

Simplified form

Review/Recap

- *Rational Expression*:
  - One polynomial divided by another
  - In other words, a fraction with a polynomial on top and another on the bottom
  - $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are both non-zero polynomials

- Fraction Arithmetic - Multiplying
  1. Simplify each fraction
  2. Multiply across
  3. Simplify the result

  Alternatively you can:
  1. Multiply across first
  2. Then simply the result

  But sometimes it helps to clean up before multiplying across…

- Simplifying
  - Cancelling means "dividing out common **factors**"
  - Factor before cancelling … always
  - This mean you ***CANNOT*** cancel in situations like this: $\frac{x+3}{x}$ because in the numerator x is a term not a factor

Homework

Pg 380, #3-24